Global Mittag-Leffler synchronization fractional complex value neural networks

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ABSTRACT

This paper introduces the use of the separation method to solve the fractional complex-valued time-varying delay neural network. The number domain is divided into the real number domain and complex-valued domain, and the Lyapunov direct method is used to study it so that the drive-response system can achieve global Mittag-Leffler synchronization under a suitable controller. It is particularly emphasized that the controller adopts an adaptive control scheme, and then by constructing an appropriate Lyapunov function proof, and then using theoretical analysis, such as fractional-differential inequality, Li's rule, etc. Sufficient criteria for global Mittag-Leffler synchronization are available, as well as some sophisticated analysis techniques. Eventually, numerical simulation is used, and then the simulation is carried out. By simulating its behavior, it is proved that our theoretical analysis and results are correct.

PRELIMINARIES

Model formulations

A driving system established by it is:

$$D_t^\alpha x_i(t) = -d_i(x_i(t))x_i(t) + \sum_{j=1}^{n} a_{ij}(x_j(t))f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(x_j(t))g_j(x_j(t-\tau_j(t))) + I_i, i, j = 1, 2. \quad (1)$$

The response system can be derived from the above formula as:

$$D_t^\alpha y_i(t) = -d_i(y_i(t))y_i(t) + \sum_{j=1}^{n} a_{ij}(y_j(t))f_j(y_j(t)) + \sum_{j=1}^{n} b_{ij}(y_j(t))g_j(y_j(t-\tau_j(t))) + I_i + u_i(t). \quad (2)$$

The Caputo derivative of the error system can be obtained from the above formula as

$$\begin{cases} D_t^\alpha z_i^R(t) = D_t^\alpha (y_i^R(t) - x_i^R(t)), \\ D_t^\alpha z_i^I(t) = D_t^\alpha (y_i^I(t) - x_i^I(t)). \end{cases} \quad (3)$$

B.Control scheme

$$\begin{cases} u_i(t) = -\lambda_i z_i(t) - l_i sign(z_i(t)) - \xi_i(t)\left[sign(z(t))|z_i(t-\tau_i(t))|\right] \\ D_t^\alpha \xi_i(t) = \eta|z_i(t-\tau_i(t))|. \end{cases} \quad (4)$$

In consequence, the control error system (3) can be modified as

(5) [illegible]

(6) [illegible]

MAIN RESULTS

Theorem 3.1. Assumes the existence of constants and satisfying for

(7) [illegible]

Therefore, system (1) and system (2) can be synchronized to the scale factor after a period, which is global Mittag-Leffler synchronization.

Proof. Under the influence of the controller (4), a Lyapunov function can be assumed.

$$V(t) \le \left(\sum_{i=1}^{n}|z_i^R(t)| + \sum_{i=1}^{n}\frac{1}{2\eta}(\xi_i(t)-\varphi)^2\right) + \left(\sum_{i=1}^{n}|z_i^I(t)| + \sum_{i=1}^{n}\frac{1}{2\eta}(\xi_i(t)-\varphi)^2\right). \quad (8)$$

Based on the stability theorem and comparison principle of linear fractional systems with multiple delays, in the framework of set-valued mapping and differential inclusion theory, some new criteria to ensure the synchronization of driven response networks are obtained:

$$\begin{aligned} D_t^\alpha V(t) &\le \sum_{i=1}^{n}\left\{-h_1\left[|z_i^R(t)| + |z_i^I(t)|\right] - h_2\left[|z_i^R(t-\tau_i(t))| + |z_i^I(t-\tau_i(t))|\right]\right\} \\ &\le -h_1\sum_{i=1}^{n}V(t) - h_2\sum_{i=1}^{n}V(t-\tau_i(t)). \end{aligned} \quad (9)$$

On the basis of Razumikhin's method and some lemmas, we have

$$V(t) \le \delta\|V(t_0)\|\left[E_\alpha(-\lambda t^\alpha)\right]^\gamma, \quad t \ge 0 \quad (10)$$

Both the system (1) and the system (2) are finally synchronized under the action of the controller (4), and it is Mittag-Leffler synchronization

NUMERICAL SIMULATIONS

The driving system is:

(11) [illegible]

The response system is:

(12) [illegible]

The state trajectories of the drive system and the response system are shown in Figures 1 and 2. The horizontal coordinate represents time, and the vertical coordinate is the state trajectory. The state track sends changes over time.

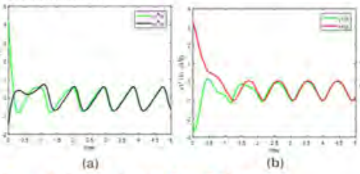

Fig. 1.Complex-valued domain state trajectories of

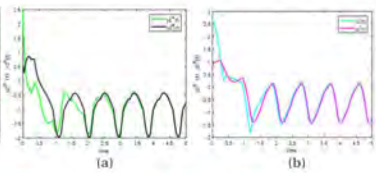

Fig. 2.Complex-valued domain state trajectories of

The state trajectories of the drive system and the response system are shown in Figures 1 and 2. The horizontal coordinate represents time, and the vertical coordinate is the state trajectory. The state track sends changes over time.

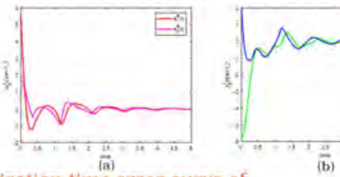

Fig. 3.Synchronization time error curve of

The synchronization error convergence behavior of and is shown in Fig. 3, (a) is the convergence behavior in the real number domain, and (b) is in the complex-valued domain.

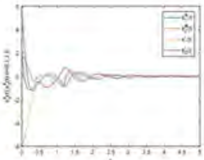

Fig. 4.Time comparison chart of the overall error of

As shown in Figure 4, it can be found that the convergence behavior trajectories of and slowly converge to around zero with time. Both systems (11) and (12) can achieve the above global Mittag-Leffler synchronization.

CONCLUSIONS

In this whole article, using the strategy method of adaptive control, fractional Leibniz rule, and Razumikhin type method, the sufficient conditions to achieve global Mittag-Leffler synchronization under a control system (4) are given. The real number domain and the complex number domain are separated from the number domain, and the two are analyzed separately, and then the effectiveness of the method is confirmed by the numerical simulation results. In the future, different controllers can be appropriately designed to achieve synchronization according to the requirements of other fractional-order systems, for example, or the number domain can be extended to the quaternion domain. Or in the next work, we aim to achieve synchronization under synovial control or impulse control. These issues have room for further research.

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